Blind Identification of Filter Length in Separation of Convolutive Signal Mixtures

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***Abstract*— Blind identification is the process of recovering a set of signals from only a set of their convolutive mixtures. This has typical applications in biomedical engineering, audio and civil engineering. Many of the existing solutions assume some *a priori* knowledge, e.g. the number of sources to recover and the length of the mixing filters, however in practical use these are typically unknown.**

This paper introduces a general method for blind estimation of the length of the mixing filters from a set of convolutive mixtures. The method is based on "A Blind Source Separation Technique Using Second Order Statistics" by *Belouchrani et al.*, where recovery of the sources is performed by an orthogonalisation of the observations followed by a de-rotation. In the proposed method for filter length estimation, we use the property that the orthogonalisation step will fail if the filter is overestimated.

The method has shown to be effective when the observations are noiseless, and less effective when observation noise is introduced. Numerical simulations are provided to illustrate this.

***Index Terms*— bss, convolution.**

I. Introduction

In medical tests involving electrical measurement of muscle activity (EMG), the recorded signals are often contributions from several different muscles i.e. a measurement around the forearm would have contributions from all the muscles controlling the fingers and wrist. As the parameters for the mixing are typically unknown, a science branch regarding "blind identification" has developed.

The mixing is not necessarily done as a linear operation, as delays and distortions can easily occur, which will result in a signal behaviour approximable by FIR filters, also known as a FIR MIMO system. This will henceforth be known as the convolutive case.

Many studies have been made on blind source separation of instantanous mixtures. The primary resource for this paper being [1] by Belouchrani et al have proposed a statistical method for separation of a linear mixing of signals (Second Order Blind Identification, SOBI). The method uses symmetric eigenvalue decomposition to recover the sources and requires that the sources are independant, stationary and have a temporal structure. This is similar to the method presented in Georgiev et al. [6]. Parra et al. [9] proposes a method using the generalized eigenvalue decomposition, which allows for separation of non-stationary sources. Blind source separation is also a branch of independant component analysis (ICA), see [5]. Studies of the conditions for blind separation to be possible includes [7], by Cao.

The SOBI method is is extended to the convolutive case in [2], also by Belouchrani et al. and further extended with non-stationary sources and blind deconvolution in [3] by Debiolles. Other solutions for the convolutive mixtures include higher order statistics as in [8] by Obradovic.

With this in regard, determination of the FIR filter order is interesting as the filter order is essentially unknown and concurrently an unknown factor in the blind separation. This paper focuses mainly on the derivation of the linear and convolutive SOBI algorithm continued with the derivation of the order determination and tests of this.

II. Method

A. SOBI for Instantaneous Mixtures

The SOBI algorithm [1] concerns separation of source signals when only their noisy linear *instantaneous* mixtures are observed. The relation between the N source signals and the M sampled observation signals is depicted on figure 1 and can be modelled as

$$\mathbf{x}(n) = \mathbf{y}(n) + \mathbf{n}(n) = \mathbf{A}\mathbf{s}(n) + \mathbf{n}(n) \tag{1}$$

where $\mathbf{x}(n) = [x_1(n), x_2(n), \ldots, x_M(n)]^{\mathrm{T}}$ is a vector of observations, $\mathbf{s}(n) = [s_1(n), s_2(n), \ldots, s_N(n)]^{\mathrm{T}}$ is a vector of source signals, $\mathbf{n}(n) = [n_1(n), n_2(n), \ldots, n_M(n)]^{\mathrm{T}}$ is a vector of observation noise realisations and $\mathbf{A}$ is an $M \times N$ mixing matrix, i.e. the transfer function between sources and observations. The transformation $\mathbf{A}$ can be interpreted geometrically as shown in figure 2(a).

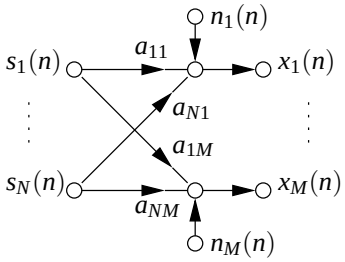

Fig. 1. Graphical representation of the mixing principle.

It is assumed that the source signals are stationary and spatially white (independant) from which it follows that the auto correlation matrix is diagonal, with the auto covariance $\rho_i(\tau)$ of each source i on the diagonal

$$\mathbf{R}_{\mathbf{ss}}(\tau) = E\left[\mathbf{s}(n)\mathbf{s}^{\mathrm{T}}(n+\tau)\right] = \mathrm{diag}[\rho_1(\tau), \ldots, \rho_N(\tau)] \tag{2}$$

where τ is a time lag in number of samples.

The noise signals are assumed to be both temporally and spatially white and also to have equal variance, σ^2. The auto correlation matrix of the noise is then given by

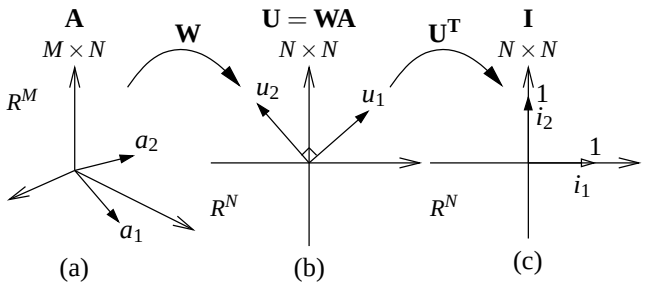

Fig. 2. Geometrical interpretation of $\mathbf{A}$, $\mathbf{U}$ and $\mathbf{I}$, which corresponds to the transformation of $\mathbf{s}(n)$ in the signals $\mathbf{x}(n)$, $\mathbf{z}(n)$ and $\mathbf{p}(n)$ respectively with $M = 3$ and $N = 2$.

$$\mathbf{R_{nn}}(\tau) = E\left[\mathbf{n}(n)\mathbf{n}^{\mathrm{T}}(n+\tau)\right] = \sigma^2\delta(\tau)\mathbf{I} \tag{3}$$

where $\delta(\tau)$ is the Kronecker delta. It is furthermore assumed that the noise signals are uncorrelated with the source signals.

The mixing matrix $\mathbf{A}$ is assumed to have full column rank, which implies $M \geq N$ and $rank(A) = N$.

Because the exchange of a fixed scalar between a source i and all observations can be either a part of the mixing matrix $\mathbf{A}$ or the source itself $s_i(n)$, the gain in a source can not be identified. Therefore it is assumed that the source signals all have unit variance, i.e. $\mathbf{R_{ss}}(0) = \mathbf{I}$. This implies

$$\mathbf{R_{yy}}(0) = E\left[\mathbf{A}\mathbf{s}(n)\mathbf{s}^{\mathrm{T}}(n)\mathbf{A}^{\mathrm{T}}\right] = \mathbf{A}\mathbf{A}^{\mathrm{T}} \tag{4}$$

The auto correlation of the observations is derived as

$$\begin{aligned}
\mathbf{R_{xx}}(\tau) &= \mathbf{A}\mathbf{R_{ss}}(\tau)\mathbf{A}^{\mathrm{T}} + \sigma^2\delta(\tau)\mathbf{I} \\
\mathbf{R_{xx}}(\tau) &= \mathbf{A}\mathbf{A}^{\mathrm{T}} + \sigma^2\mathbf{I}, \quad \tau = 0 \qquad (5) \\
\mathbf{R_{xx}}(\tau) &= \mathbf{A}\mathbf{R_{ss}}(\tau)\mathbf{A}^{\mathrm{T}}, \quad \tau \neq 0 \qquad (6)
\end{aligned}$$

It can be seen from (1) that the source signals can be determined from the observations by multiplication with the pseudo inverse of the mixing matrix, $\mathbf{A}^{\#}$. The vector $\mathbf{p}(n)$ denotes the result from this multiplication:

$$\mathbf{p}(n) = \mathbf{A}^{\#}\mathbf{x}(n) = \mathbf{s}(n) + \mathbf{A}^{\#}\mathbf{n}(n) \tag{7}$$

The following will show that the transformation $\mathbf{A}$, which has been performed on the sources, can be interpreted as a shear transformation and a rotation, see figure 3(a). To acquire the source signals the observations must be *a)* whitened, see figure 3(b) and *b)* rotated to axis alignment, see figure 3(c).

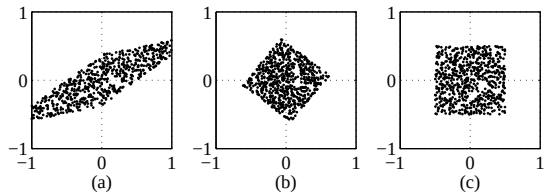

Fig. 3. Data representation of $\mathbf{x}(n)$, $\mathbf{z}(n)$ and $\mathbf{p}(n)$ plottet for 600 samples. The observations are assumed noiseless therefore the three graphs corresponds also to $\mathbf{A}\mathbf{s}(n)$, $\mathbf{U}\mathbf{s}(n)$ and $\mathbf{s}(n)$. The data is plottet for $M = 2$ and $N = 2$. The sources are drawn as white noise signals only to make the figure easy to read, as this violates one of the assumptions.

a) Whitening: To make the signal part of the observations spatially white, the signal part of the observations is multiplied with a whitening matrix $\mathbf{W}$, which satisfies

$$\mathbf{R_{WyWy}}(0) = \mathbf{I} = E\left[\mathbf{W}\mathbf{y}(n)\mathbf{y}(n)^{*}\mathbf{W}^{\mathrm{T}}\right] \tag{8}$$

The vector $\mathbf{z}(n)$ denotes the whitened observations:

$$\mathbf{z}(n) = \mathbf{W}\mathbf{x}(n) = \mathbf{W}\mathbf{A}\mathbf{s}(n) + \mathbf{W}\mathbf{n}(n) \tag{9}$$

The whitening implies that the space is changed from observation space $\mathbf{R}^M$ to signal space $\mathbf{R}^N$ as shown in figure 2(a-b), i.e. the whitening matrix will have dimensions $N \times M$. The matrix $\mathbf{W}$ can be determined from $\mathbf{R_{xx}}(0)$ since, using (4) and (5), the auto correlation of the signal part of the observation is given by

$$\mathbf{R_{yy}}(0) = \mathbf{R_{xx}}(0) - \sigma^2\mathbf{I} \tag{10}$$

where the noise variance σ^2 can be estimated as an average of the $M - N$ lowest eigenvalues of $\mathbf{R_x}(0)$ if it is not known a priori. Since $\mathbf{R_{xx}}(0)$ is symmetric and $\mathbf{A}$ has full rank $\mathbf{R_{yy}}(0)$ can take the diagonal form $\mathbf{R_{yy}}(0) = \mathbf{P}\mathbf{D}\mathbf{P}^{\mathrm{T}} - \sigma^2\mathbf{I} = \mathbf{P}(\mathbf{D} - \sigma^2\mathbf{I})\mathbf{P}^{\mathrm{T}}$ as a result of the spectral theorem [4], where $\mathbf{D}$ is a diagonal matrix with real eigenvalues of the matrix $\mathbf{R_{xx}}(0)$ and $\mathbf{P}$ is a matrix with the corresponding orthonormal eigenvectors as columns. Using the whitening definition from (8) a closed form equation for the matrix $\mathbf{W}$ is derived:

$$\begin{aligned}
\mathbf{I} &= \mathbf{R_{WyWy}}(0) \\
&= \mathbf{W}\mathbf{R_{yy}}(0)\mathbf{W}^{\mathrm{T}} \\
&= \mathbf{W}\mathbf{P}\left(\mathbf{D} - \sigma^2\mathbf{I}\right)\mathbf{P}^{\mathrm{T}}\mathbf{W}^{\mathrm{T}} \\
&= \mathbf{W}\mathbf{P}\left(\mathbf{D} - \sigma^2\mathbf{I}\right)^{\frac{1}{2}}\left(\mathbf{D} - \sigma^2\mathbf{I}\right)^{\frac{1}{2}}\mathbf{P}^{\mathrm{T}}\mathbf{W}^{\mathrm{T}} \\
&= \left(\mathbf{W}\mathbf{P}\left(\mathbf{D} - \sigma^2\mathbf{I}\right)^{\frac{1}{2}}\right)\left(\mathbf{W}\mathbf{P}\left(\mathbf{D} - \sigma^2\mathbf{I}\right)^{\frac{1}{2}}\right)^{\mathrm{T}} \\
\mathbf{I} &= \mathbf{W}\mathbf{P}\left(\mathbf{D} - \sigma^2\mathbf{I}\right)^{\frac{1}{2}} \\
&\Updownarrow \\
\mathbf{W} &= \left(\mathbf{D} - \sigma^2\mathbf{I}\right)^{-\frac{1}{2}}\mathbf{P}^{\mathrm{T}} \qquad (11)
\end{aligned}$$

b) Rotation to axis alignment: The matrix transformation $\mathbf{WA}$ from (9) is in the following denoted $\mathbf{U}$ and can be interpreted as the residue transformation after the observations are whitened. It follows that the mixing matrix can be determined if $\mathbf{U}$ is known,

$$\mathbf{U} = \mathbf{W}\mathbf{A} \Leftrightarrow \mathbf{A} = \mathbf{W}^{\#}\mathbf{U} \tag{12}$$

where # denotes the pseudo inverse. Using the whitening definition

$$\mathbf{W}\mathbf{R_{yy}}(0)\mathbf{W}^{\mathrm{T}} = \mathbf{W}\mathbf{A}\mathbf{A}^{\mathrm{T}}\mathbf{W}^{\mathrm{T}} = (\mathbf{W}\mathbf{A})(\mathbf{W}\mathbf{A})^{\mathrm{T}} = \mathbf{I} \tag{13}$$

it can be seen that $\mathbf{U}$ is a complex unitary $N \times N$ matrix. Since the matrix $\mathbf{U}$ is unitary the transformation can be interpreted as a rotation in R^N. Assuming $\mathbf{U}$ is known the sources can be separated by de-rotating $\mathbf{z}(t)$:

$$\mathbf{p}(n) = \mathbf{U}^{-1}\mathbf{z}(n) = \mathbf{U}^{\mathrm{T}}\mathbf{z}(n) = \mathbf{s}(n) + \mathbf{U}^{\mathrm{T}}\mathbf{W}\mathbf{n}(n) \tag{14}$$

This is shown geometrically in figure 2(c). The matrix $\mathbf{U}$ can be determined from the auto correlation of the whitened observations for a time lag different from zero, which yields

$$\mathbf{R_{zz}}(\tau) = \mathbf{W}\mathbf{R_{xx}}(\tau)\mathbf{W}^{\mathrm{T}} = \mathbf{U}\mathbf{R_{ss}}(\tau)\mathbf{U}^{\mathrm{T}}, \quad \forall\,\tau \neq 0 \quad (15)$$

by using (6), (9) and (12). From the spectral theorem it follows that the auto correlation of the whitened sources is diagonalized by the unitary matrix $\mathbf{U}$ [4]. However the $\hat{\mathbf{U}}$ that can be obtained as the eigenvectors of $\mathbf{R_{zz}}(\tau)$ will be only essentially equal to $\mathbf{U}$ [1], meaning that the rows of $\hat{\mathbf{U}}$ can be a permutation of the rows in $\mathbf{U}$. In practice this means that the order of the separated signals can not be determined.

Indeterminacy is furthermore introduced if the eigenvalues of $\mathbf{R_{zz}}(\tau)$ are not distinct, which they will not always be if the spectral shapes of the source signals are similar. A robust solution to this issue is to use a set of k time lags, and determine the matrix $\mathbf{U}$ that minimizes the total sum of squares of the off-diagonal entries in the matrices $\mathbf{U}^{\mathrm{T}}\mathbf{R_{zz}}(\tau_i)\mathbf{U}$ for $i = 1, \ldots, k$. This operation is referred to as Joint Diagonalization (JD).

B. SOBI for Convolutive Mixtures

The SOBI algorithm [1] can be extended to solve a convolutive system (SOBICM), in which constants a_{ij} of figure 1 is replaced with FIR filters H_{ij} of order L. The method is described in [3] and [2].

The general system notation can be written as a generalisation of the instantaneous case

$$\begin{aligned} \mathbf{x}(n) &= \mathbf{H}(0)\mathbf{s}(n) + \mathbf{H}(1)\mathbf{s}(n-1) + \ldots & (16)\\ \ldots &+ \mathbf{H}(L)\mathbf{s}(n-L) + \mathbf{n}(n) & (17)\end{aligned}$$

This notation reduces the convolutive model to that of the instantaneous, examining each delayed part as a new source and observation. This is achieved by introducing an integer L', ensuring fulfillment of $ML' \geq N(L+L')$, and using delayed versions of $\mathbf{X}$ and $\mathbf{S}$ as new representations.

$$\begin{aligned} \mathbf{S_i}(n) &= [s_i(n), .., s_i(n-(L+L')+1)] & (18)\\ \mathbf{X_j}(n) &= [x_j(n), .., x_j(n-L'+1)] & (19)\\ \mathbf{N_j}(n) &= [n_j(n), .., n_j(n-L'+1)] & (20)\\ \mathbf{A_{ij}} &= \begin{bmatrix} h_{ij}(0) & \cdots & h_{ij}(L) & \cdots & 0 \\ 0 & \ddots & & \ddots & \vdots \\ \vdots & \ddots & & \ddots & 0 \\ 0 & \cdots & h_{ij}(0) & \cdots & h_{ij}(L) \end{bmatrix} & (21)\end{aligned}$$

where $i = 1, .., N$ and $j = 1, .., M$. The resultant elements in the convolutive system is the column vectors

$$\begin{aligned} \mathbf{S}(n) &= [\mathbf{S_1}, .., \mathbf{S_N}]^{\mathrm{T}}, \text{ of size } N(L+L') \times 1 & (22)\\ \mathbf{X}(n) &= [\mathbf{X_1}, .., \mathbf{X_M}]^{\mathrm{T}}, \text{ of size } ML' \times 1 & (23)\\ \mathbf{N}(n) &= [\mathbf{N_1}, .., \mathbf{N_M}]^{\mathrm{T}}, \text{ of size } ML' \times 1 & (24)\end{aligned}$$

and the new mixing matrix is represented as $\mathbf{A}$

$$\mathbf{A} = \begin{bmatrix} \mathbf{A_{11}} & \cdots & \mathbf{A_{1N}} \\ \vdots & \ddots & \vdots \\ \mathbf{A_{M1}} & \cdots & \mathbf{A_{MN}} \end{bmatrix} \quad (25)$$

where $\mathbf{A}$ is full column rank and of size $ML' \times N(L+L')$. The convolutive system is thereby similar to the instantaneous:

$$\mathbf{X}(n) = \mathbf{A}\mathbf{S}(n) + \mathbf{N}(n) \quad (26)$$

The derivation of the convolutive SOBI method is analogous to the linear mixture case, recognizing that block-matrices occur as a consequence of correlation between delayed sources. This results in the auto-correlation matrix $\mathbf{R_{SS}}(\tau)$ of $\mathbf{S}$ being block diagonal with size $N(L+L') \times N(L+L')$:

$$\mathbf{R_{SS}}(\tau) = \begin{bmatrix} \mathbf{R_{S_1S_1}}(\tau) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{R_{S_NS_N}}(\tau) \end{bmatrix} \quad (27)$$

with the zeroes occurring due to $\mathbf{S_i}(t), \forall i$ being spatially white.

Whitening of $\mathbf{X}(n)$ is achieved the same way as for the instantaneous SOBI, see equation (11).

Rotation to axis alignment is achieved by examining the whitened observations and disregarding noise

$$\mathbf{R_{ZZ}}(\tau) = \mathbf{W}\mathbf{R_{XX}}(\tau)\mathbf{W}^{\mathrm{T}} \quad (28)$$

and acknowledging that

$$\begin{aligned} \mathbf{R_{XX}}(\tau) &= \mathbf{A}(\mathbf{R_{SS}}(\tau))\mathbf{A}^{\mathrm{T}} & (29)\\ \mathbf{R_{ZZ}}(\tau) &= \mathbf{WA}(\mathbf{R_{SS}}(\tau))\mathbf{A}^{\mathrm{T}}\mathbf{W}^{\mathrm{T}} & (30)\end{aligned}$$

Using $\mathbf{W}\mathbf{R_{XX}}(0)\mathbf{W}^{\mathrm{T}} = \mathbf{I}$ from the whitening and inserting (29) for $\tau = 0$ yields

$$\begin{aligned} \mathbf{I} &= \underbrace{\mathbf{WA}\mathbf{R_{SS}}(0)^{\frac{1}{2}}}_{\mathbf{U}} \underbrace{\mathbf{R_{SS}}(0)^{T\frac{1}{2}}\mathbf{A}^{\mathrm{T}}\mathbf{W}^{\mathrm{T}}}_{\mathbf{U}^{\mathrm{T}}} & (31)\\ \mathbf{WA} &= \mathbf{U}\mathbf{R_{SS}}(0)^{-\frac{1}{2}} & (32)\end{aligned}$$

Inserting (32) into the expression of the whitened observations (30) it is seen that $\mathbf{U}$ is the eigenvectors of the decomposition of $\mathbf{R_{ZZ}}(\tau)$.

$$\mathbf{R_{ZZ}}(\tau) = \mathbf{U}\left(\mathbf{R_{SS}}(0)^{-\frac{1}{2}}\mathbf{R_{SS}}(\tau)\mathbf{R_{SS}}(0)^{-T\frac{1}{2}}\right)\mathbf{U}^{\mathrm{T}} \quad (33)$$

The matrix $\mathbf{U}$ is thus unitary and rotates the whitened observations to axis alignment.

The diagonalisation of $\mathbf{R_{ZZ}}(\tau)$ could be done for any value of τ, assuming that $\mathbf{R_{ZZ}}(\tau)$ has distinct eigenvalues in each block. To achieve better robustness a joint block-diagonalisation (JBD) is used.

The whitening and rotation to axis alignment produces the expression

$$\hat{\mathbf{S}}(t) \approx \mathbf{R}^{\mathrm{T}}\mathbf{R_{SS}}(0)^{-\frac{1}{2}}\mathbf{S}(t) \quad (34)$$

as found in [3, p. 19], where $\mathbf{R}$ is the permutation matrix signifying the indeterminancy of sequence.

The final step in the global algorithm is blind deconvolution. Blind deconvolution is necessary as SOBICM only partially reconstruct the sources. The new sources consist of a source

component, but still filtered by a FIR filter. In [3] A. Debiolles implement an algorithm called Noise Subspace Method. This algorithm exploit the orthogonality between a noise and a signal subspace, and estimates the FIR filter in the least mean square sense.

C. Filter length estimation

In the SOBICM algorithm the filter order is required as a parameter since the order determines how many times each observation signal must be repeated in order for $\mathbf{A}$ to have full column rank. In the signal model for SOBICM underestimating L corresponds to overlooking the last coefficients in the impulse responses of the filters in $\mathbf{A}$. Overestimating L corresponds to introducing trailing zeros in the impulse responses. If under- or overestimation of L can be determined blindly, the correct filter length can also be determined.

From (21) the mixing matrix $\mathbf{A}$ has the size $ML' \times N(L+L')$, these dimensions is denoted $Q \times P$ in the following. Because of the structure of $\mathbf{A}$, it is seen that when trailing zeros are added to each impulse response, $\mathbf{A}$ will have columns with only zeros and will therefore no longer have full column rank. In particular the rank will be $P' = P - NL_o$, where L_o is the number of overestimated coefficients. This is also reasonable from looking at (21), where the last $\mathbf{H}$s will have rank zero. The fact that $\mathbf{A}$ is not full rank is a violation of one of the assumptions made, and the algorithm will not separate the sources efficiently.

In the mixing model the impulse responses of $\mathbf{A}$ will have trailing zeros. This does not affect the filtering or mixing, but blind separation of the mixed signals is impossible, i.e. $\mathbf{A}$ can not be determined. So instead of $\mathbf{A}$ the autocorrelation matrix of the observations is used.

For simplicity the observation noise is neglected for now:

$$\mathbf{R_{XX}}(0) = \mathbf{A}\mathbf{R_{SS}}(0)\mathbf{A}^{\mathrm{T}} \tag{35}$$

Some symmetric matrix $\mathbf{B}$ of dimension $m \times m$ is said to be positive semidefinite if $\mathbf{v}^{\mathrm{T}}\mathbf{B}\mathbf{v} \geq 0$ for all vectors $\mathbf{v}$ of dimension $1 \times m$. Hence the autocorrelation matrix of the (delayed) sources is always positive semidefinite:

$$\begin{aligned} \mathbf{v}^{\mathrm{T}}\mathbf{R_{SS}}(0)\mathbf{v} &= \mathbf{v}^{\mathrm{T}} E\left[\mathbf{S}(n)\mathbf{S}(n)^{\mathrm{T}}\right]\mathbf{v} \\ &= E\left[\mathbf{v}^{\mathrm{T}}\mathbf{S}(n)\mathbf{S}(n)^{\mathrm{T}}\mathbf{v}\right] \\ &= E\left[(\mathbf{v}^{\mathrm{T}}\mathbf{S}(n))(\mathbf{v}^{\mathrm{T}}\mathbf{S}(n))^{\mathrm{T}}\right] \\ &= E\left[(\mathbf{v}^{\mathrm{T}}\mathbf{S}(n))^2\right] \geq 0 \end{aligned} \tag{36}$$

However we will assume that in practice $\mathbf{R_{SS}}(0)$ is positive definite. Since $\mathbf{R_{SS}}(0)$ is positive definite and has full rank, the rank of $\mathbf{R_{XX}}(0)$ is given as the rank of $\mathbf{A}$, using also the property that $rank(\mathbf{A}) = rank(\mathbf{A}^{\mathrm{T}})$. Thus if $rank(\mathbf{R_{XX}}(0)) = P$ then $\mathbf{A}$ has full column rank. Similarly if $rank(\mathbf{R_{XX}}(0)) < P$ then $\mathbf{A}$ has not full column rank and L is overestimated.

Still $\mathbf{R_{XX}}(0)$ is symmetric and therefore it can be diagonalized. The effect on the eigenvalues of $\mathbf{R_{XX}}(0)$ is similar to the above: If $\mathbf{A}$ has rank P then $\mathbf{R_{XX}}(0)$ has P nonzero eigenvalues and if $\mathbf{A}$ has a rank $P' < P$ then $\mathbf{R_{XX}}(0)$ has P' nonzero eigenvalues.

As a summary the following sentences will hold if one of them holds:

1) Mixing matrix $\mathbf{A}$ has rank $P' < P$
2) $\mathbf{R_{XX}}(0)$ has rank P'
3) $\mathbf{R_{XX}}(0)$ has P' nonzero eigenvalues
4) The filter order L is overestimated

Hence a sufficient and necessary condition for blindly stating that L is overestimated is that $\mathbf{R_{XX}}(0)$ has less than P nonzero eigenvalues. Similarly if $\mathbf{R_{XX}}(0)$ has exactly P nonzero eigenvalues then L either correctly estimated or underestimated. This follows from the fact that an underestimation of L is assumed not to affect the rank of $\mathbf{A}$ as an overestimation certainly does. When L is overestimated and the number of nonzero eigenvalues is $P' < P$, the correct filter order can be calculated from solving the equation

$$\begin{aligned} P' &= N(L+L') \\ &= N(L + \lfloor LM/(M-N) \rfloor + 1) \end{aligned} \tag{37}$$

where L is the correct filter order and $\lfloor \cdot \rfloor$ is the floor operator.

When noise is introduced on the observations equation (35) changes to

$$\mathbf{R_{XX}}(0) = \mathbf{A}\mathbf{R_{SS}}(0)\mathbf{A}^{\mathrm{T}} + \mathbf{R_{NN}}(0) \tag{38}$$

where $\mathbf{R_N}(0)$ is a diagonal matrix with the noise variance on the diagonal. It follows that all eigenvalues of $\mathbf{R_X}(0)$ will be nonzero as the noise variance, σ^2, is added to each of the eigenvalues of $\mathbf{A}\mathbf{R_S}(0)\mathbf{A}^{\mathrm{T}}$. The eigenvalue matrix of $\mathbf{R_X}(0)$ will have the diagonal form

$$\begin{bmatrix} \lambda + \sigma_N^2 & 0 & 0 \\ 0 & \sigma_N^2 & 0 \\ 0 & 0 & \hat{\sigma}_N^2 \end{bmatrix} \tag{39}$$

where $\lambda + \sigma_N^2$ is a subdiagonal holding entries $\lambda_i + \sigma_N^2, i = 1, .., P'$, σ_N^2 is a subdiagonal with $P - P'$ entries of the noise variance and $\hat{\sigma}_N^2$ is a subdiagonal of $M - N$ entries which is used to estimate the noise variance.

The whitening matrix, $\mathbf{W}$, which is calculated as shown in (11), uses always the first P eigenvalues of $\mathbf{R_{XX}}(0)$ assuming that $\mathbf{A}$ has full rank. When $\mathbf{A}$ does not have full rank $\mathbf{R_{XX}}(0)$ will have $P - P'$ equal eigenvalues. Since $\mathbf{W}$ is constructed from the largest P eigenvalues which includes the $P - P'$ equal ones, the rows of $\mathbf{W}$ will be linearly dependent. This will result in faliure of the whitening step, i.e. when L is overestimated the whitened observations will still be spatially correlated. This is easily detected using the correlation coefficient matrix of $\mathbf{WX}(n)$ to time lag zero.

The error that is introduced in the whitening of the observations when L is overestimated is utilized in generation of an index, ρ, that is used to track how much L is overestimated. The index is calculated as

$$\rho = \frac{\sum |\Phi_{\mathbf{WX}}| - \mathbf{I}}{Q(Q-1)} \tag{40}$$

where $\Phi_{\mathbf{WX}}$ is the auto correlation coefficient matrix of $\mathbf{WX}(n)$ and $Q(Q-1)$ is the number of offdiagonal entries. The index $\rho(L)$, will be a monotonically increasing function with a steep ascent between the correct filter length and the successive filter length.

The following provides an outline of calculation of the index $\rho(L)$ is:

1) Calculate $L' = \lfloor LN/(N-M) \rfloor + 1$.
2) Generate $\mathbf{X}(n)$ from L' delayed versions of each observation according to (23).
3) Calculate eigenvectors and eigenvalues of $\mathbf{R}_{\mathbf{XX}}(0)$ and generate $\mathbf{W}$.
4) Calculate the correlation coefficient matrix of $\mathbf{WX}(n)$ by normalizing $\mathbf{WR}_{\mathbf{XX}}(0)\mathbf{W}^{\mathrm{T}}$.
5) Calculate the index $\rho(L)$ from (40).

III. Simulations

The sources used in the simulations are band pass filtered white noise sources of 1000 samples. Thereby the sources are stationary and spatially uncorrelated as required by the algotithm, and are further more different in their PSD, which improves the separability.
In all simulations the number of sources and the number of observations corresponds to $M \geq N(L+1)$[1].

A. Performance Index

The performance index, used in verification of the algorithm, expresses how well the sources are reconstructed. The index have basis in the correlation between the original and the estimated source. To use all sources $\mathbf{S}$ and estimates $\widehat{\mathbf{S}}$ together in a clearly defined index, all sources and estimates have to be normalised. This normalisation is performed by making all the variances equal to one:

$$\sigma_{\mathbf{S}} = \sigma_{\widehat{\mathbf{S}}} = 1 \tag{41}$$

With $\sigma_{\mathbf{S}}$ and $\sigma_{\widehat{\mathbf{S}}}$ normalised the goodness is expressed as

$$\text{Performance index} = \frac{\sum_{i=1}^{N} R_{\mathbf{s}_i \widehat{\mathbf{s}}_i}(\tau)}{N} \tag{42}$$

where $R_{\mathbf{S}\widehat{\mathbf{S}},i}(\tau)$ is the crosscorrelation function for the i'th source and estimate, and τ is the time lag in the between $\mathbf{S}$ and $\widehat{\mathbf{S}}$. The time lag should equal zero, as this corresponds to deconvoluting a FIR filter of correct order.

B. Influence of filter length estimate on performance index

The filter length, number of sources and time lags have to be known a priori to full fill the model Equation 26. The number of sources is assumed to be know or can be estimated. The time lags used for the joint block diagonalisation can be chosen a priori, but the selection cannot be assumed to not influence the performance index. To prove whether or not an incorrect estimate of filter length decreases or increases the performance index, a simulation where the estimated filter length is varied from two to seven have been performed. For each estimated filter length 50 different subset of time lags have been chosen, to ensure that the selection of time lags does not influence on the simulation. The filter length, number of sources are held constant and the number of observations full fill $ML' \geq N(L+L')$.

[1]Derived from $M \cdot L' \geq N(L+L')$ by letting $L' = 1$. This corresponds not using delayed observations.

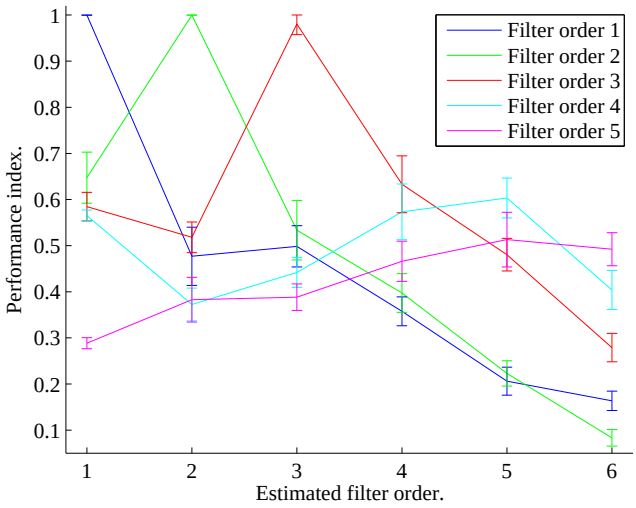

Fig. 4. Results of simulation where the filter length is held constant and estimated filter length is varied. Each line represent a filter length as a mean of the performance index for the 50 measurements for each estimated filter length. The vertical lines are a 95 % confidence interval.

Figure 4 shows the results from the simulation. Each line represent a filter length as the mean of the performance indexes for the 50 measurements for each estimated filter length. The vertical lines are a 95 % confidence interval. The results show, that the performance index for the filter lengths two,three and four maximises when the correct filter length is estimated. For these lengths the performance decreases when the length is incorrect estimated. For the filter lengths five and six, it cannot be statically determined if the performance index increases or decreases when the correct filter length is estimated. The filter lengths two, three and four have a performance index over 0.95 which can be approximated to the same as a complete separation. For the filter lengths five and six have a performance index for the correct filter length respectively below 0.64 and 0.58. This shows, that the performance index decreases when the filter length increases as concluded in [3]. It can be concluded, that the performance index maximises for filter lengths from two through four when the correct filter length is estimated. For the filter lengths five and six it cannot be concluded. The performance index decreases, when the filter length increases.

C. Blind filter order identification index.

The blind filter order index ρ is propose as a blind identification index in Section II-C see Equation 40. The simulation has to prove whether or not there is a relationship between the index and the filter order. If there is an association, the simulation has to show which effect noise has on the index. The simulation is performed as four sub simulations, where each simulation represent respectively a SNR at 90, 60, 30, and 0 dB. Each simulation is performed with two sources and $NL' \leq M(L+L')$ observations. The simulation measure the index ρ for each filter order form first to 15'th order. Each measurement is performed with five

different sets of sources and ten different filters of 7'th order. Figure 5 show the results from the simulation. The line in each sub figure is respectively the mean of the measurements and the mean of the derivative of the measurements. The pluses are a 95 % confidence interval, under the assumption the measurements are normal distributed. The results show, that the index ρ increase, when the filter order is overestimated, in the case where the SNR is 60 and 90 dB. In the cases where the SNR is 0 and 30 dB, the index ρ starts to increase before the filter order is overestimated. It have to be noticed, that the scale of the axises are different. This shows, that the index ρ increases more rapidly when the SNR increases.
In table I, the spread of the mean between the sub simulations for each filter order is shown. The table show, that the values of the mean for all measurements from order 1 to 5 is approximately equal. When the filter order is overestimated the spread increases rapidly and then settles. It can be

Filter order	**1-5**	**6-10**	**11-15**
σ_m	1.6799e-17	2.4331e-15	1.1215e-07
σ_m	3.4422e-17	8.0862e-14	1.3301e-07
σ_m	4.0405e-17	1.2711e-08	1.5924e-07
σ_m	1.3707e-17	4.973e-08	1.75e-07
σ_m	2.1696e-16	7.738e-08	1.8805e-07

TABLE I
THE TABLE SHOWS THE SPREAD OF THE MEAN BETWEEN THE SUB SIMULATIONS FOR EACH FILTER ORDER.

conclude from the simulation, that the index ρ increases when the filter order is overestimated. When the SNR is increased the index ρ increases more rapidly. The index is for all SNR approximately equal when the filter length is underestimated. The index ρ settles, when the order is overestimated.

D. Order Estimation Method

To illustrate the effectiveness of the proposed method for estimation of filter order, an algorithm is implemented and tested for SNR of 90 dB. The algorithm uses the mean of the absolute values in the off-diagonal of $\Phi_{\mathbf{WX}}$ and calculates the ratio, or slope, between the present and previous value, $\rho(L_e)$ for L_e being the present estimated order. The recently calculated ratio is compared to the previous $\rho(L_e - 1)$, and a threshold is used for deciding whether the filter order is overestimated:

$$\frac{\rho(L_e)}{\rho(L_e - 1)} > threshold \tag{43}$$

If this ratio is larger than $threshold$ the correct order is $L_e - 1$. This algorithm gives an indication of how well the method performes, but an implementation with dynamically adaptive $threshold$ is expected to improve performance considerably in relation to observation noise. This could eventually be implemented by use of the previously obtained slope-ratio, i.e. for $L_e - 1$.

The test is run for 30 different sets of sources, and for each set of sources ten different sets of filters. The results are seen in figure 6.

From the upper plot in figure 6 it is seen that the estimation for all L is correct in more than 93% of the cases. For the order estimates of one to eight it is correct in more than 98% of the cases. For the higher orders it becomes more likely to under- and overestimate. Looking at the lower figure, this corresponds to more than 93% of the points lie on a straight line intersecting $(L_i, L_{e,i})$ for $i = \{1, 2, .., 15\}$. It is seen that the wrong estimates corresponds to under- and overestimation of one order. Relating this to figure 5,

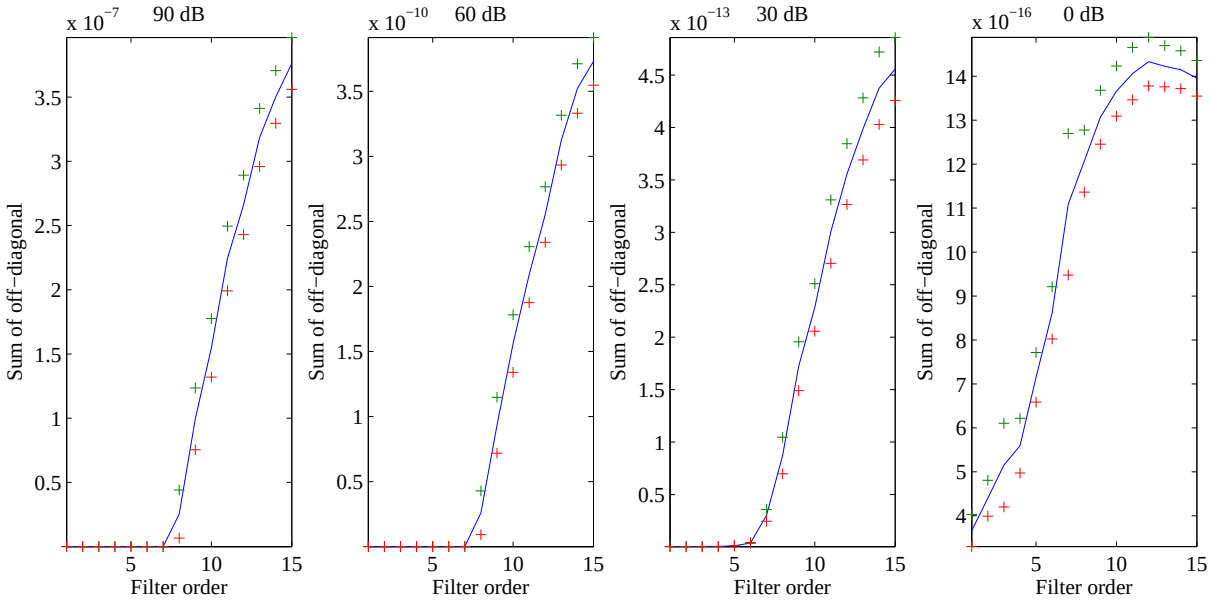

Fig. 5. The figure show four sub figures. Each sub figure represent a SNR for the simulation. The sub figures is ρ as a function of the filter order. The simulations is perform which 2 sources and $NL' \leq M(L + L')$ observations. The filter order is seven for all simulations. The lines represent the mean of the measurements of each order and the pluses is a 95 % confidence interval. It is assumed that the measurements is normal distributed.

it makes sense, as it is observed that the derivative slightly change before the correct order is obtained. Thereby the implemented algorithm wrongly estimates the correct order, and this is explained as a consequence of the static $threshold$.

Thereby, the method is shown to function in a case of 90 dB SNR, using the ratio between present and previous mean of the absolute off-diagonal of $\Phi_{\mathbf{WX}}$ together with a static threshold for decision making.

IV. Discussion

The purpose of this paper was to present a method for blind filter length estimation with regard to the convolutive mixing case. A method examining the whitening step and concluding on length estimation correctness has been presented and tested, albeit the actual selection algorithm is a quick test-implementation. The results are that the method works and is very general in the sense that the signals must fulfill the assumptions, but the method itself is implementation independant. It is furthermore seen that the estimation of filter length is important as incorrect estimation yields a poor separation.

Examining actual performance tests of the algorithm shows a hit rate between 95%-100% for orders of 15 or less, far exceeding the 3rd or 4th order of which the SOBICM implementation works correctly. Introducing noise produces a higher sum of the off-diagonal entries, making the order selection criteria more ambiguous. This is clearly an area where the method needs improvement and is thus suggested for further study.

As a sidenote, the selection of lags for the joint-block-diagonalisation is seen to be important for the performance of the source separation. This effect was discovered as a curiosity but clearly an area of much depth and is thus also recommended for further study.

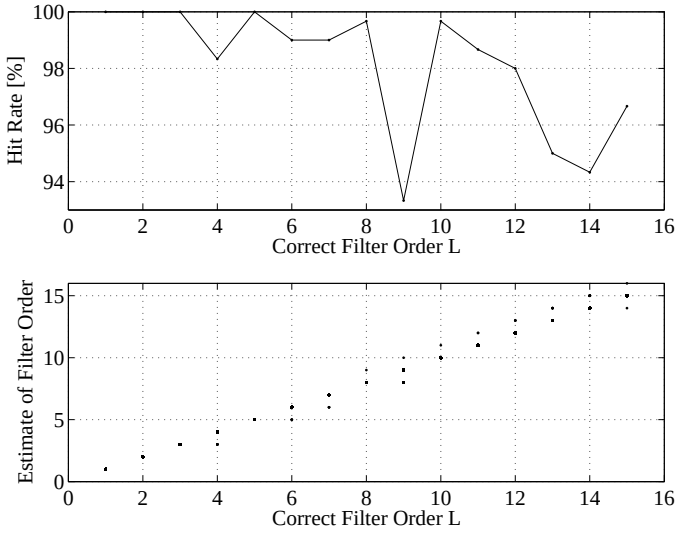

Fig. 6. Illustration of erder estimation method with SNR of 90 dB, by use of mean of absolute value of the off-diagonal of $\Phi_{\mathbf{WX}}$. In the upper plot, the percentage of which the proposed algorithm estimated correctly is denoted Hit Rate (93 to 100 $\%$), and the resolution is 0.33%. In the lower plot the estimated order is shown. Both are plotted versus the correct filter order $L = \{1, 2, .., 15\}$, and $threshold = 175$.

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